Cellular automaton model considering the velocity effect of a car on the successive car

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In this paper we present a cellular automata model for one-lane traffic flow. The update rules of velocity of a car depend not only on the positions of this car and the car ahead of it, but also on the velocities of the two cars. Using computer simulations we obtain some basic qualitative results and the fundamental diagram of the proposed model. In comparison with those of the existing models in the literature, we find that the fundamental diagram of the proposed model is more consistent with the results measured in the real traffic, and the model is able to reproduce some relevant macroscopic states that are found in the real traffic flow but cannot be predicted by the existing models.

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I. INTRODUCTION

The investigation of models describing traffic flow on freeways began in the early 1950s. Traditionally, there are two types of models, microscopic and macroscopic models. Macroscopic models regard the whole traffic flow as a flow of continuous medium based upon a continuum approach (see e.g., Ref. [1]). Microscopic models simulate behavior details of each car to get evolution rules (see, e.g., Ref. [2]). In the early 1990s, the rapid development of computer capacity allowed a type of relatively new microscopic traffic flow models, e.g., cellular automata (CA) to display its high practical importance. Cellular automata are dynamical models that have many but discrete degrees of freedom. In fact there is a threefold discreteness in cellular automata: discrete space, discrete time, and discrete number of states [3,4]. When applied to traffic research, CA use cellular state to describe the position and velocity of each car, update every cell state with rules educed by practical traffic experiences, and get the whole system's dynamical evolvement and final steady result. Compared with continuum models, CA traffic models are much simpler and more convenient for computer simulations. The most important aspect is that CA models can model the complexities of nonlinear characters in traffic problems, and offer more intuitive physical images.

Presently, there are two basic CA models that describe single-lane traffic flow, the Nagel-Schreckenberg (NS) model [5] and the Fukui-Ishibashi (FI) model [6]. Both of them are defined on a one-dimensional lattice of L sites with periodic boundary conditions. Each site is either occupied by a vehicle, or is empty. The velocity of each vehicle is an integer between zero and $V_{\max}$. If $X_{(i,t)}$ denotes the position of the ith car at time t, the position of the car ahead of it at time t is $X_{(i+1,t)}$. With these notations, the system evolves according to synchronous rules given by

$$X_{(i,t+1)}=X_{(i,t)}+V_{(i,t+1)}, \tag{1}$$

where in the NS model

$$V_{(i,t+1)}=\min\{V_{\max},V_{(i,t)}+1,d_{(i,t)}\} \tag{2}$$

and in the FI model

$$V_{(i,t+1)}=\min\{V_{\max},d_{(i,t)}\}, \tag{3}$$

where $d_{(i,t)}$, denoting the gap (number of empty sites) between cars i and $i+1$, is called the "headway" of the ith car and can be given by

$$d_{(i,t)}=X_{(i+1,t)}-X_{(i,t)}-1. \tag{4}$$

If we do not consider randomization caused by other complicated influences, these two models differ only in the velocity's acceleration rules. That is, the NS model restricts the cars to gradual accelerations while the FI model allows for abrupt increase if there is enough empty spacing ahead.

Essentially, these two basic models are analogical. Both the gradual and the abrupt acceleration update rules of the two models for the ith car velocity depend on the headway $d_{(i,t)}$, which is determined by the positions of the two successive cars, i and $(i+1)$. The $(i+1)$th car is regarded as stockstill, whereas actually it may move ahead at the same time. On the road, a driver looks at the next car and takes the information of that car as main factor in determining the velocity of his own car at the next time step. The information includes not only the position of the next car, but also its velocity, because when one car moves ahead the next car may also move along. Therefore, it is more realistic to update the ith car's velocity according to the velocities of the ith and the $(i+1)$th cars as well as their positions. For example, when the $(i+1)$th car velocity is large enough, even if the headway of the ith car is zero, the ith car can still move ahead.

The above discussions lead us to present a different single-lane CA model based on the thought that the velocity update rules depend on both the velocities and the gap of two successive cars. According to these rules, computer simulations are performed to get the basic qualitative results and the fundamental diagram of the proposed model. Compared with the two basic models, our model displays some superior characters.

The paper is organized as follows. In Sec. II we describe the proposed model and present the results of computer

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simulations. In Sec. III, we discuss the advantages of our model. At last we reach the final conclusion in Sec. IV.

II. MODEL AND SIMULATION RESULTS

A. Velocity update rule

In the following we introduce the proposed model under the same condition as the two basic models. The notation is also similar. At each discrete time step $t \rightarrow t+1$, the velocity of the ith car is updated according to the following rules:

$$v = \min\{V_{\max}, V_{(i,t)}+1, d_{(i,t)}+V'_{(i+1,t+1)}\}, \tag{5}$$

where $V'_{(i+1,t+1)}$ is an imaginary velocity of the $(i+1)$th car at the $(t+1)$ time step, given as

$$V'_{(i+1,t+1)} = \min\{V_{\max}-1, V_{(i+1,t)}, \max\{0, d_{(i+1,t)}-1\}\}. \tag{6}$$

We call $V'_{(i+1,t+1)}$ "virtual velocity" which is obtained by applying the velocity update rule of the NS model to the $(i+1)$th car and considering the random delay. It is the least value that the (i+1)th car can move by any possibility at the $(t+1)$ time step when its velocity varies according to the NS model rules and when random delay is considered.

It can be seen that the third parameter of the minimal function at the right hand side of Eq. (5) is different from Eq. (2) of the NS model in that the virtual velocity of the $(i+1)$th car is added to the headway. The sum of the headway and the virtual velocity is the allowable moving area of ith car at the $(t+1)$ time step. Therefore, we derive v by comparing this sum with the maximum velocity and restricting the cars to gradual accelerations.

After v is obtained, randomization is applied: with probability f, the velocity of each car (if greater than zero) is decreased by 1. Thus $V_{(i,t+1)} = v$, or $V_{(i,t+1)} = v-1$.

B. Simulation and results

In the simulation, the length of a site corresponds to 7.5 m on a real road, one automaton time step is 1 s and the velocity unit is roughly 27 km/h. It is assumed that $V_{\max}$ equals 5, which implies a maximum velocity of 135 km/h, just as the normal free-flow speed in the real traffic. The total number of sites, denoted by L, is assumed to be 2000, and density ρ is defined as $\rho = N/L$, where N is the number of cars. Initially, N cars are located randomly on the sites and the velocity of each car is designated by an integer randomly chosen from zero to $V_{\max}$. From left to right, the N cars are numbered orderly from 1 to N. In each time step that follows, the velocities of the N cars are renewed according to the velocity updating rules, with the probability $f = 0.3$, and then the cars move forwards simultaneously. The periodic boundary condition is applied when necessary. The data are collected after the time evolution reaches to the 20 000th step.

When plotted in a space-time diagram the CA evolution displays free flow at a low density while start-and-stop waves at certain high density. Figures 1 and 2 show the different final states of the system in two different densities.

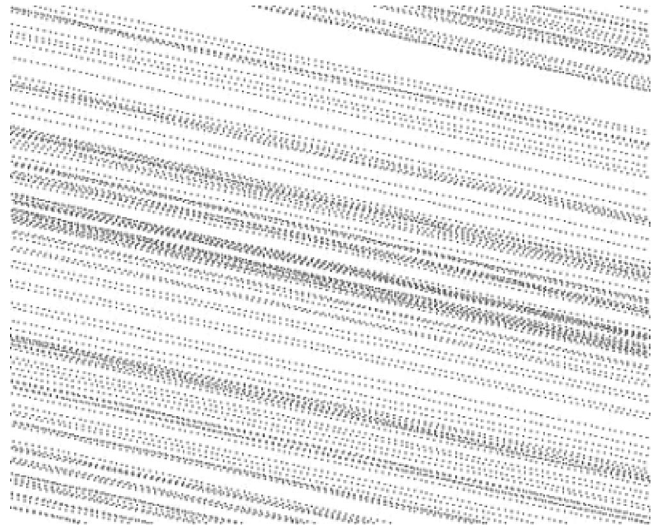

FIG. 1. $\rho = 0.05$, free flow. The horizontal direction is space and the vertical direction (down) is (increasing) time.

In addition we can obtain some other macroscopic states under certain special conditions, such as a shock in the traffic, see Fig. 3.

III. SUPERIOR CHARACTERS

A. Fundamental diagram

Fundamental diagram is one of the most important criteria that evaluate the transit capacity of the simulation result for a one-lane traffic flow model. The fundamental diagram of the NS model has been found to have a nonlinear tendency that is consistent with the results measured in the real traffic but the critical point departs from the real traffic. In fact, both the maximum flux and the critical density are about a factor of 2 lower than the real traffic measurements. Therefore, the transit capacity of the NS model is much lower than the actual capacity in the real traffic (see [7]).

However, we obtain a different fundamental diagram under the proposed model with the same simulation conditions

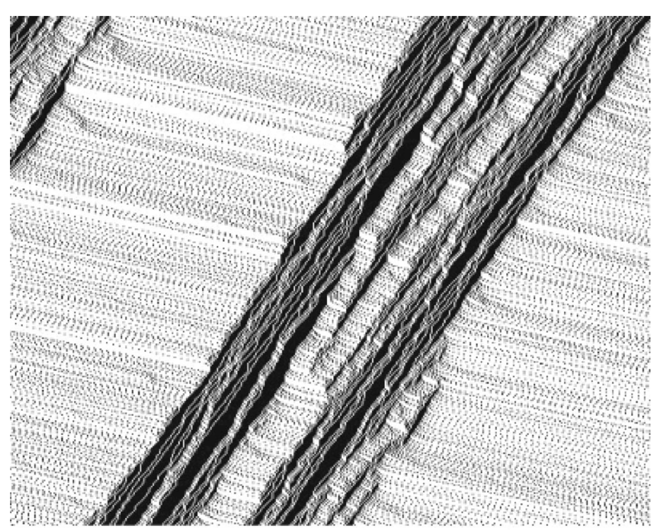

FIG. 2. $\rho = 0.25$, the start-and-stop waves.

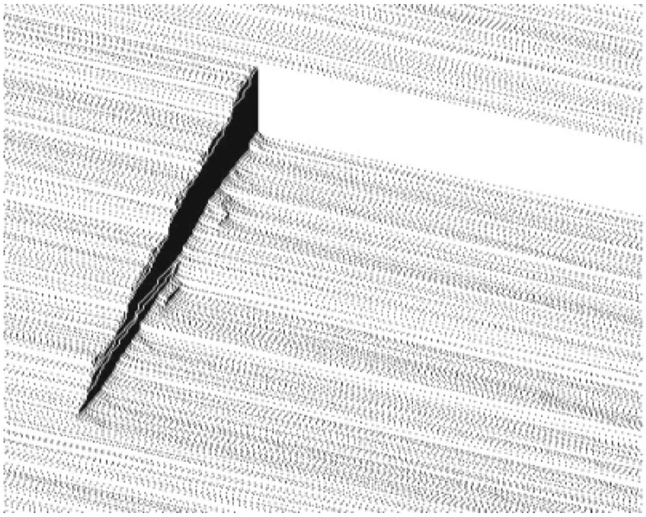

FIG. 3. $\rho=0.10$. After the system reaches the steady state, at certain site cars are forbidden to pass for 30 time steps. The black triangle behind that site is the result. The shape of that triangle depends on the average density and the acceleration rules.

as that of the NS model, see Fig. 4. This new fundamental diagram is much closer to the real measurement than the one under the NS model.

As we can see from Fig. 4, the three curves have similar characteristics. However, in the critical point area, the results of our proposed model and the measurements are closer, both larger than that of the NS model. The maximum flux of our model is 0.61, closer to the measurement value of 0.65, in comparison with 0.47 of the NS model. Therefore, the transit capacity of our model is more realistic. This improvement can be explained as follows: The NS model takes the $(i+1)$th car as a fix barrier in front of the ith car, so the movement of the ith car is limited to the headway. However, the $(i+1)$th car can actually move ahead at a certain speed at the same time, thus the ith car can move forward further than the headway. As a result, the average speed of the N cars in the NS model is smaller than that in the real traffic. That is why the average flux of the NS model is smaller than the real

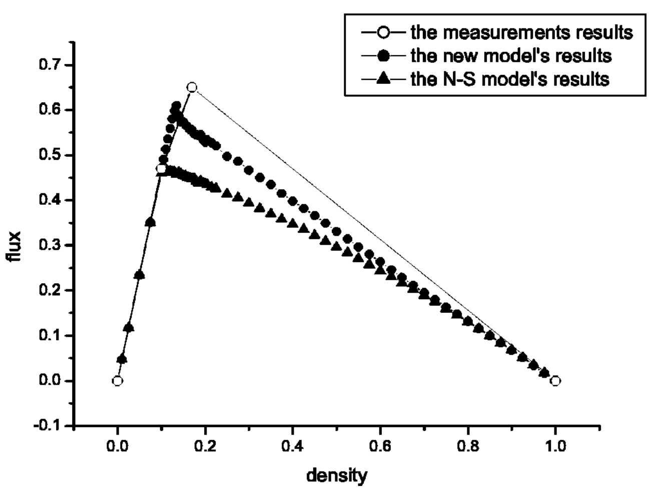

FIG. 4. Fundamental diagrams.

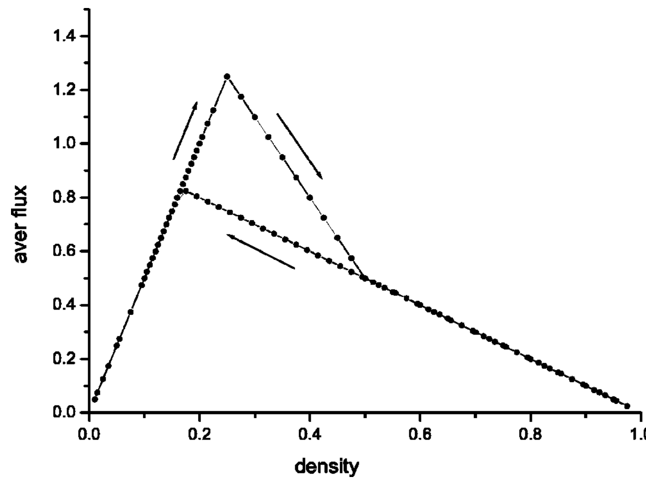

FIG. 5. The hysteresis loop of the proposed model when $f=0$.

measurement as indicated in Fig. 4. However, in the proposed model, the update rules of the ith car velocity take into account the movement of the $(i+1)$th car. That is, we first use the virtual velocity to estimate the velocity of the $(i+1)$th car conservatively by using the velocity update rules of the NS model, then add this virtual velocity to the headway. The summation is assumed to be the area in which the ith car can move in $t+1$ time, which is wider than that of the NS model. Thus the average speed and the flux of the model improve. The update rules of velocity are more consistent with the real traffic conditions and the resulting fundamental diagram is closer to the real measurement curve.

B. The hysteresis

Under a specific condition, the given model can reproduce an important character of traffic flow, ''hysteresis'' (see [8,9]), which cannot be produced by the NS model.

Figure 5 depicts the fundamental diagram for $f=0$ in the interesting range of densities. It can be seen that two branches of the fundamental diagram exist above the critical density. The upper branch is calculated by adding cars to a

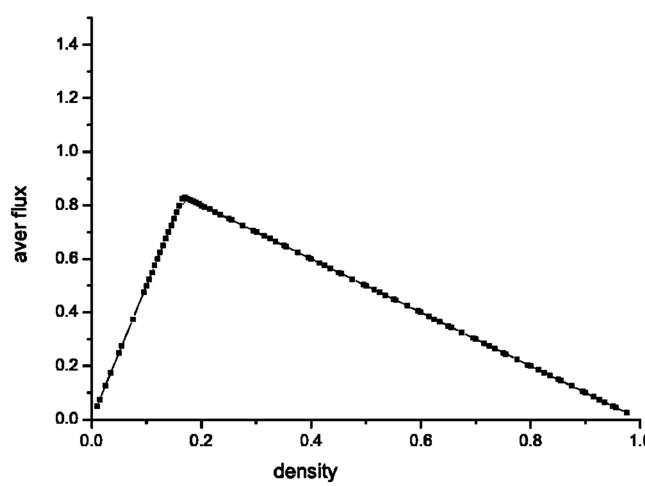

FIG. 6. The results of the NS model under the same condition as in Fig. 5.

homogeneous state, while the lower branch is calculated by removing cars from a jammed state and allowing the system to relax after the intervention. In this way a hysteresis loop can be traced (arrows in Fig. 5).

As shown in Fig. 5 the system is bistable in a certain range of density. Starting from an initial inhomogeneous configuration, the system will have some jams that are never sorted out. The steady state in this case is an inhomogeneous mixture of the jam-free region and higher density jammed regions. Clearly, these jammed regions decrease the average flux in the system, thus we get the lower branch of the loop. As for the initial homogeneous configuration without any jam, since all motion is deterministic in this state, the steady state will also be free of jams and the flux will still be a linearly increasing function of the density until the ith car cannot keep moving at$V_{\max}$ as long as the sum of the headway and the virtual velocity of the $(i+1)$th car is less than $V_{\max}$. If each headway is 3, the system reaches the maximum flux, which is obviously different from the flux under an initial inhomogeneous configuration. That is the explanation for the upper branch.

When using the NS model to perform the same procedure as described above, we cannot get any new branch of the fundamental diagram. As displayed in Fig. 6, the fundamental diagrams under the two conditions overlap.

IV. CONCLUSIONS

In this paper we present a CA model for one-lane traffic flow. By introducing ''virtual velocity,'' we have considered the influences of the motion of a car on the car following it. The given model can reproduce some common characteristics of the real traffic, such as the ''start-and-stop'' waves and the shock in traffic flow. The fundamental diagram of the proposed model is more consistent with the real traffic measurement than that of the NS model, which does not take the effect of velocity into account. A special phenomenon in the real traffic—''the hysteresis''—can also be reproduced by the given model, which cannot be explained by the NS model.

It is evident from the above discussion that, our model has some distinct superiorities, and therefore is worth further investigation.

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